The Didactics of Elementary Mathematics for the Blind Child in an Inclusive Context

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The current context of Inclusive Education in Portugal, brought by Law 116 of September 13, 2019 has as guiding principles, equity, inclusion, diversity, personalization, flexibility and parental involvement. This view requires the teacher to use active and differentiated work methodologies in the classroom, which promote everyone's learning. In particular, in Essential Learning for Mathematics (Mathematical Curriculum) in the early years, respect for the principles of equity and quality are presented as objectives of the teaching of Mathematics, contemplating relevant and sustainable learning for all.

The current challenge for the mathematics teacher, who has blind students in his/her group or class, consists of creating didactic strategies and differentiated work methodologies that enhance meaningful learning for all. In this context, mathematical communication takes on an expression of enormous relevance. This article aims to show some differentiated examples of resources and methodological strategies focused on mathematical communication, which can be implemented within a classroom context. The developed strategies integrate specific material resources for exploration through the haptic system and methodological strategies focused on mathematical communication that facilitate personal autonomy and the full inclusion of all. Its operationalization made it possible to observe an effective participation of all students and, in particular, of blind students in the mathematics classroom.

Because being present is not enough, it is necessary to participate actively…

Key words: Communication, Mathematics, Inclusive Education, Didactics, Essential Learning, Pedagogical Differentiation, Perception, Haptic System

Introduction

The challenge for Maths teachers who nowadays have blind students in their group/class is to come up with different didactic and methodological strategies for that can result in significant learning for all. The Communication of Mathematics becomes of great importance. Through this communication, students can grasp the basic notions of mathematics. The objective of this article is to show the importance in using manipulative material resources when teaching mathematics to all students and especially blind ones. This article also aims to show how important it is for the teacher to apply the Universal Design Learning (UDL) as methodological strategy in the classroom. To do this, an example of a Lesson Plan using the instruments and resources according to the methodology described (UDL), will be presented.

Description of Study

Case Study

Applied investigation methodology was based on the qualitative study of a Case Study design. The qualitative research is used to identify studies whose data are essentially of a qualitative nature and therefore difficult to put into statistics. The purpose of this research is not to answer questions and/or check hypotheses. The qualitative research is used to analyze descriptive details related to the sample behavior in a specific ecological context, being sometimes referred to as naturalist (Bogdan & Biklen, 1994). The procedure consists in the integration of the researcher during considerable amount of time in the natural environment where the samples can be found. The nature of the results is a descriptive one and the data collected are images and/or words. There is greater interest in the procedure than in the results themselves. Data Analysis is an inductive process. The planning is obtained through research. The meaning given to the actions is of great importance to the understanding of the environment in which the research is carried out. (Bogdan & Biklen, 1994).

Two Schools from the district of Lisbon were included in this study. Both are reference schools for Low Vision and Blindness. School 1 is a Private Social Welfare Institution and School 2 is a Public Reference School for Low Vision or Blind Students.

This study included two small second grade classes. School 1 had 15 pupils while School 2 had 16. Each of these classes included a blind student.

School 1	School 2
Albano	Alana
Alexandra	Bruce
Carlos	Iana
Beatriz	Iris
José	Ivo
Gaspar	Lia
Hugo	Lysandro
Maria	Patrício
Silvério	Policarpo
Tiago	Rosa
Felisberto	Rui
Rui	Raya
Cecília	Sandra
Luísa	Tiago
Rita	Irina
	Miguel

Participants in the study: School 1 - Private Social Welfare Institution (Lisbon): 9 boys and 6 girls (one blind student – Luísa); School 2 – Public Reference School for Low Vision or Blind Students (Lisbon): 8 boys and 8 girls one blind student – Irina).

All students are aged between 7 – 8 years old, except for Alana who is 13 years old.

Sample dimension is of 31, 2nd grade students.

Table 1: Students' Ficticious Names

Students Cognitive Profile and Measures to Support Learning and Inclusion

Knowing the cognitive profile of each student is essential to the teacher so that he/she can choose and apply the support measures that will allow the teaching and inclusion of the curriculum in much simpler way. Following this, two list charts containing the Cognitive Profile description of each blind student is presented as well as the support measures used for inclusion in the classroom.

Student	Cognitive Profile	Measures
Luísa (School 1)	Very cheerful, participating child who shows un extraordinary will to learn in general. Gets along well with classmates and vice-versa. Still depends on the presence of adults during recreation. Likes to carry out all the tasks given. Follows all curricular content with only adaptations concerning *Braille* resources. Uses *Braille* in class. The student has residual vision which is expected to disappear in a short/medium term, which is why the teaching reading and writing in the two ways has been undertaken, so as to recognize all letters in both ink and *Braille*.	Selective Measures: Art. 9 c) Psycho-pedagogical support with the special needs teacher three times a week in the classroom; d) Anticipation and reinforcement of learning (support from the special needs teacher and head teacher with the aim of improving reading, writing and calculation skills and the use of support technologies, namely CCTV and *Braille* machine). Specific curricular areas (paragraph d) of Art. 2): *Braille* system; Orientation and mobility; Specific information and communication technologies. Adaptations in the assessment process (Article 28): a) Diversification of data collection instruments, such as surveys, interviews, video or audio recordings; b) Text in accessible formats, namely *Braille*, tables and relief maps, DAISY, digital; d) Use of supporting products; e) Extra time given to take tests; f) Transcript of responses and g) Reading of information.

Table 2: Luisa's Cognitive Profile and Measures

These measures are defined by the Law n.º 54 of 7 July 2018, with new alterations presented by Law 116 of 13 September 2019.

Student	Cognitive Profile	Measures
Irina (School 2)	Extrovert, cooperative and with a strong sense of initiative. Popular amongst peers and interacts in a very correct and empathetic way with peers and adults. Hardworking and cooperative to carry out tasks. Always available to help her peers and adults in different activities. She learns essential subjects without difficulty, uses *Braille* literacy (reading and writing). In the classroom, she uses the *Perkins Brailler Machine* to write. She likes to explain to others how to use the specific resources for blind. She has a good communication skill (orality). Her greatest potential is in terms of reasoning, memorization and tactile dexterity (motor skills).	Selective Measures: Art. 9 c) Psycho-pedagogical support with the special needs teacher four times a week in the classroom (4 hours); d) Anticipation and reinforcement of learning (support from the special needs teacher and head teacher with the aim of improving reading, writing, calculating skills and the use of support technologies). Specific curricular areas (paragraph d) of Art. 2): *Braille* system; Orientation and mobility; Specific information and communication technologies (screen reader). Adaptations in the evaluation (Article 28): a) Diversification of data collection instruments, such as surveys, interviews, video or audio recordings; b) Listed in accessible formats, namely *Braille*, tables and relief maps, DAISY, digital; d) Use of support products; e) Additional time for taking the test; and g) Reading the information.

Table 3: Irina's Cognitive Profile and Measures

Students Irina and Luísa read and write using the *Braille* Mathematical code. They both master Portuguese and Mathematics *Braille* Code. In the classroom, they use the Perkins *Brailler* Machine.

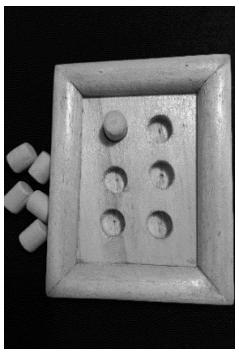


Figure 1 *Braille* Cell

Teachers begin the study of writing in *Braille* using the amplified *Braille* Cell as shown in Figure 1. The Perkins *Brailler* Machine is introduced when Pre-*Braille* skills are acquired.

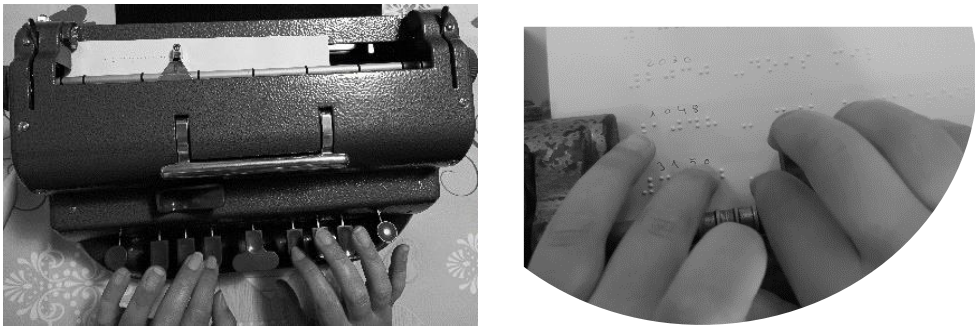


Figure 2 Perkins *Brailler* Machine

Methodology Strategies

- Integrated Table (Bloom Taxonomy and Gardner's Multiple Knowledge)

Gardner argues that everyone innately has all kinds of intelligence, although some of them are more developed in some people than in others. What is important to keep in mind is that all forms of intelligence are valid and there is not one more valuable than the other, even because, in today's competitive and complex world, most professions require the simultaneous use of several types of intelligence.

The teacher must be able to recognize which type of intelligence is most developed in each of his/her student so as to know the way they learn. This will allow the teacher to create a series of different and dynamic tasks which will make learning easier for all students.

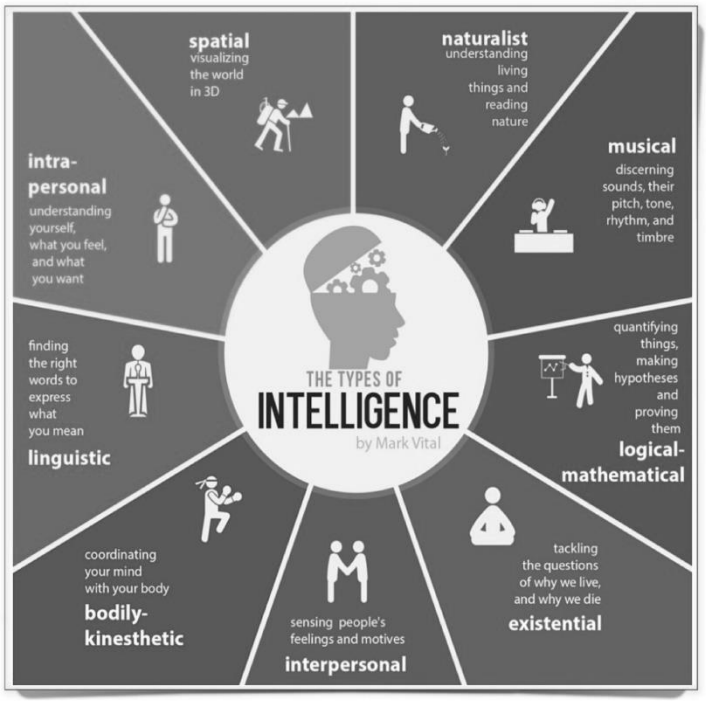


Figure 3 Gardner's Multiple Knowledge

The away the questions are asked and the nature can vary according to Bloom's Taxonomy.

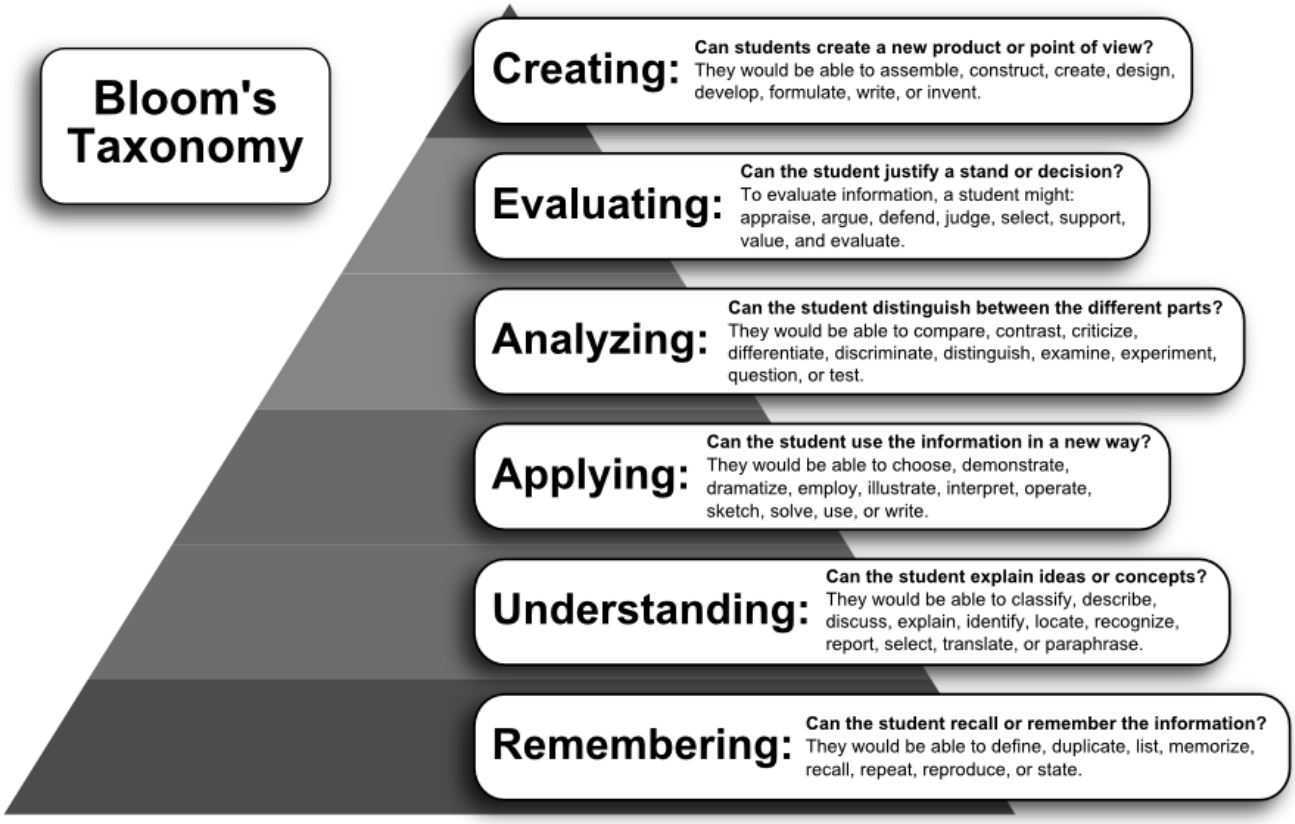


Figure 4 Bloom's Taxonomy

- Universal Design for Learning

The term Universal Design (UD) or Design Universal (DU) appeared in the 1970s after the Vocational Rehabilitation Act was approved in the United States in 1973 forbidding any type of discrimination related to handicaps. This principal was introduced in the educational procedure by the Universal Design for Learning which, in turn, introduced an approach to teaching and learning that includes proactive curriculum planning (goals, assessments, methods and materials). Planning with UDL does not take a one-size-fits-all approach; rather, it takes into account the diversity of all students. (Cast, 2014)

Figure 5 David Rose **"One Size does not fit all"**

Universal Design stems from the principle of accessibility used by architects, which implies designing environments that allow access to everyone, whatever physical and cognitive needs they may have (Edyburn, 2010; Rose & Gravel, 2010). Full access designed from beginning greater cost-benefit ratio compared to later adaptations. The fusion between esthetics and functionality was developed. "*Consider the needs of the broadest possible range of users from the beginning*" as defended by the architect Ron Mace.

The UDL became possible through the use of several neuroscientific learning principals along with principal of availability to all.

The Principals of Neurocience and its importance in a Classroom Environment:

- The brain possesses optimal periods (sensitive periods) for certain types of learning that never break down even with age. Adjust expectations and patterns of development to the specific characteristics of the age groups using integrated themes.
- New learning experiences simultaneously activate several areas of the cerebral cortex. Situations that reflect real life experiences in such a way that the new information connects itself to the previous knowledge.
- As a result of the experience, the brain modifies itself, little by little, both physiologically and structurally. Practical classes/Physical exercises actively involving the participants associate previous experiences with the present knowledge.

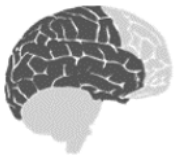
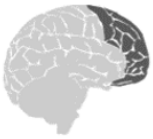
Universal Design for Learning		
Recognition Networks The "What" of Learning	Strategic Networks The "How" of learning	Affective Networks The "Why" of learning
 How we gather facts and categorize what we see, hear and read. Identifying letters, words or and author's style are recognition tasks. Present information and content in different ways **Multiple Means of Representation**	 Planning and performing tasks. How we organize and express our ideas. Writing an essay or solving a math problem are strategic tasks. Differentiate the ways that students can express what they know. **Multiple Means of Action and Expression**	 How learners get engaged and stay motivated. How they are challenged, excited, or interested. These are effective dimensions. Stimulate interest and motivation for learning. **Multiple Means of Engagement (Motivation)**

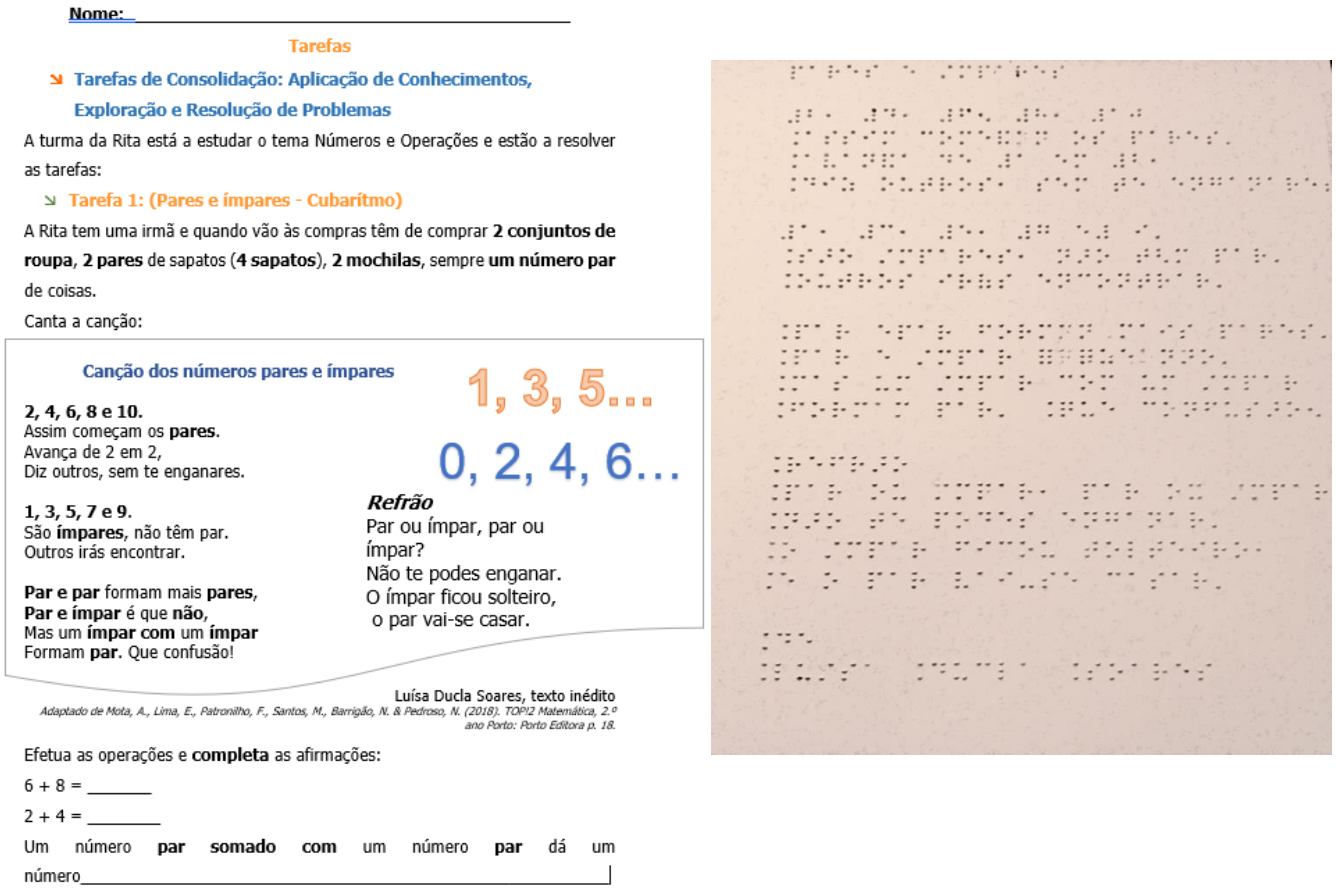
Table 4: Universal Design for Learning (UDL)

The following Table shows the task that were given. Each task was created for a 2nd grade class (Table 5).

Theme	Essential Learning (curriculum)	Learning Practices	Tasks	
			School 1	School 2
Números e Operações	Read and represent numbers in the decimal numbering system up to 1000 and identify the positional value of a digit.	Explore, analyze and interpret situations from different contexts - Mathematics learning (concepts, properties, operations, and mathematical procedures).	Task 1	Task 1 Task 2
	Identify and give examples of odd and even numbers.	Use structured and unstructured manipulative materials and other resources in problem solving and other learning tasks.		Odd and even Song Task 1
	Compare and order numbers. Calculate possible arrangements of sums, subtractions, and multiplications with and without using concrete material.	Use basic facts of operations in calculation situations, namely basic facts of addition (eg, 3+3=6, 5+5=10) and multiplication (table of 2, 3, 4, 5 and 10)	Task 2 – Problem Solving	
	Recognize unit fractions as representations of a part of a whole (unit) divided into equal parts, in different contexts.		Task 3 Exercises	Task 3 - Division
Data Organization and Processing	Communicate reasoning, procedures and results	Communicate, orally and in writing	Task 4	Task 4
Geometry and Measurement	Identify and compare geometric solids, recognizing similarities and differences, and identifying polygons (triangles, squares, rectangles, pentagons and hexagons) and circles in these solids.	Communicate using Mathematical language, orally and in writing, to describe and explain reasoning, procedures and conclusions.	Task 5	Task 5

Table 5: Tasks

To exemplify, three tasks using explorational resources will be presented. Task 1 is about numbers and operations and was used to introduce even and odd numbers. The only resources used were an even and odd Song and Task instructions. The task instructions were written in bold letters so as to be easier to understand. This task consisted in the listening and singing of an even and odd Song.



Nome: _____

Tarefas

Tarefas de Consolidação: Aplicação de Conhecimentos, Exploração e Resolução de Problemas

A turma da Rita está a estudar o tema Números e Operações e estão a resolver as tarefas:

Tarefa 1: (Pares e ímpares - Cubaritmo)

A Rita tem uma irmã e quando vão às compras têm de comprar **2 conjuntos de roupa**, **2 pares** de sapatos (**4 sapatos**), **2 mochilas**, sempre **um número par** de coisas.

Canta a canção:

Canção dos números pares e ímpares

1, 3, 5...

0, 2, 4, 6...

2, 4, 6, 8 e 10.
Assim começam os **pares.**
Avança de 2 em 2,
Diz outros, sem te enganares.

1, 3, 5, 7 e 9.
São **ímpares**, não têm par.
Outros irás encontrar.

Par e par formam mais **pares,**
Par e ímpar é que **não,**
Mas um **ímpar com** um **ímpar**
Formam **par.** Que confusão!

Refrão
Par ou ímpar, par ou ímpar?
Não te podes enganar.
O ímpar ficou solteiro,
o par vai-se casar.

Luísa Ducla Soares, texto inédito

Adaptado de Mota, A., Lima, E., Patronilho, F., Santos, M., Barrigão, N. & Pedroso, N. (2018). TOP!2 Matemática, 2.º ano Porto: Porto Editora p. 18.

Efetua as operações e **completa** as afirmações:

6 + 8 = _____

2 + 4 = _____

Um número **par somado com** um número **par** dá um número_____

Figure 6 Task1 in Ink and *Braille* Code

Here is an example given by a normovisual student.

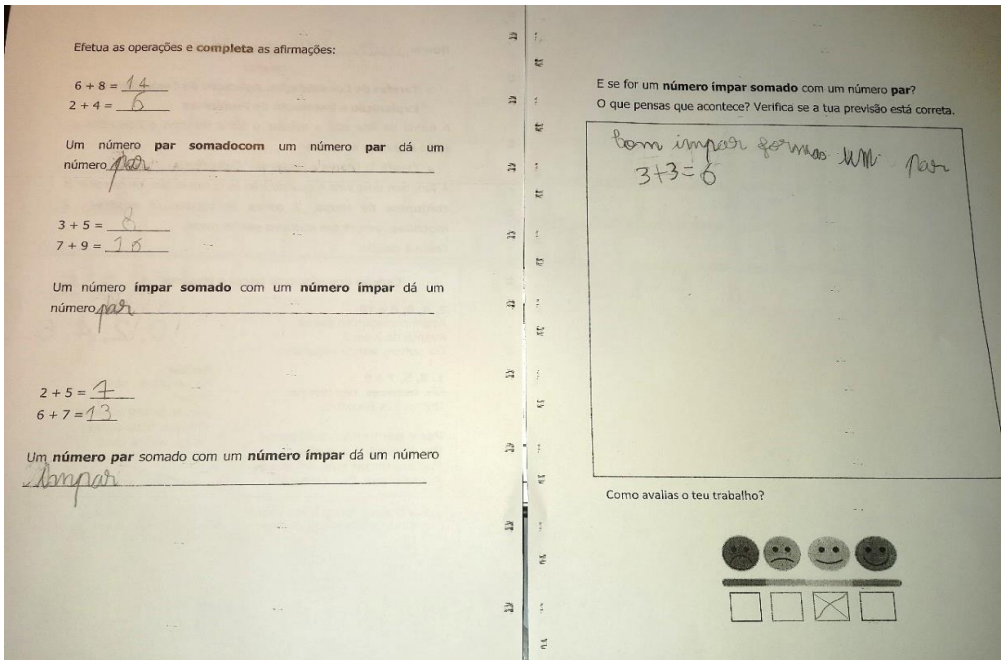


Figure 7 An example given by a normovisual student

The Multiplane was used by the blind student to solve Task1.

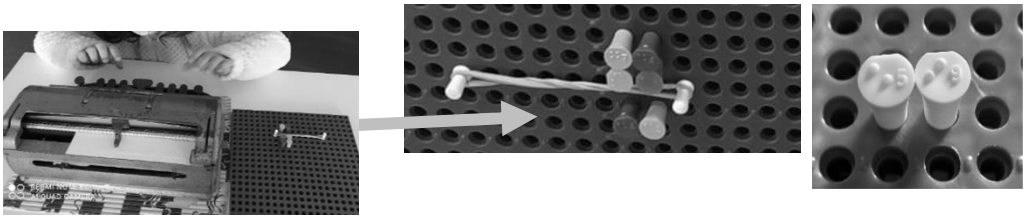


Figure 8 Example of a solution made by using a Multiplane

Another resource used to solve Algebra Calculations is a Cubarithm.

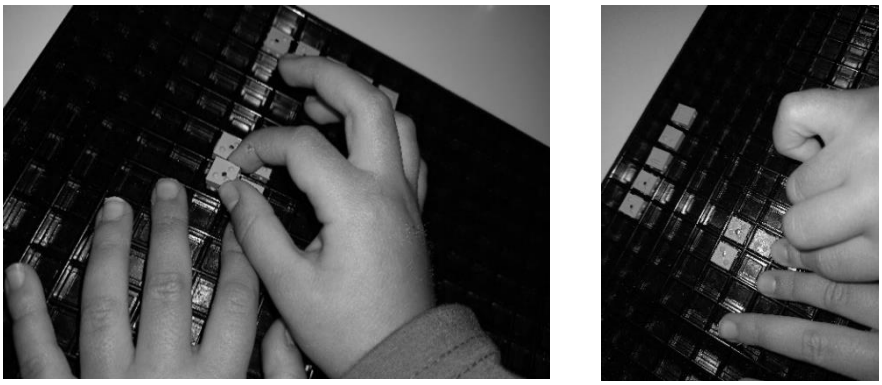


Figure 9 Using the Cubarithm

The main difference between the Multiplane and the Cubarithm is the fact that the Multiplane has numbers in ink (surface) and the Cubarithm only use the *Braille* Code. This makes teachers prefer the Multiplane of it is easier to communicate between teacher/blind student. In School 1, Problem Solving and Calculations were made using Multibasic Calculaters and Luísa successfully completed the task.

Figure 10 The using of Multibasic Calculaters by Luísa

Normovisual colleagues drew and colored Multibasic Calculaters.

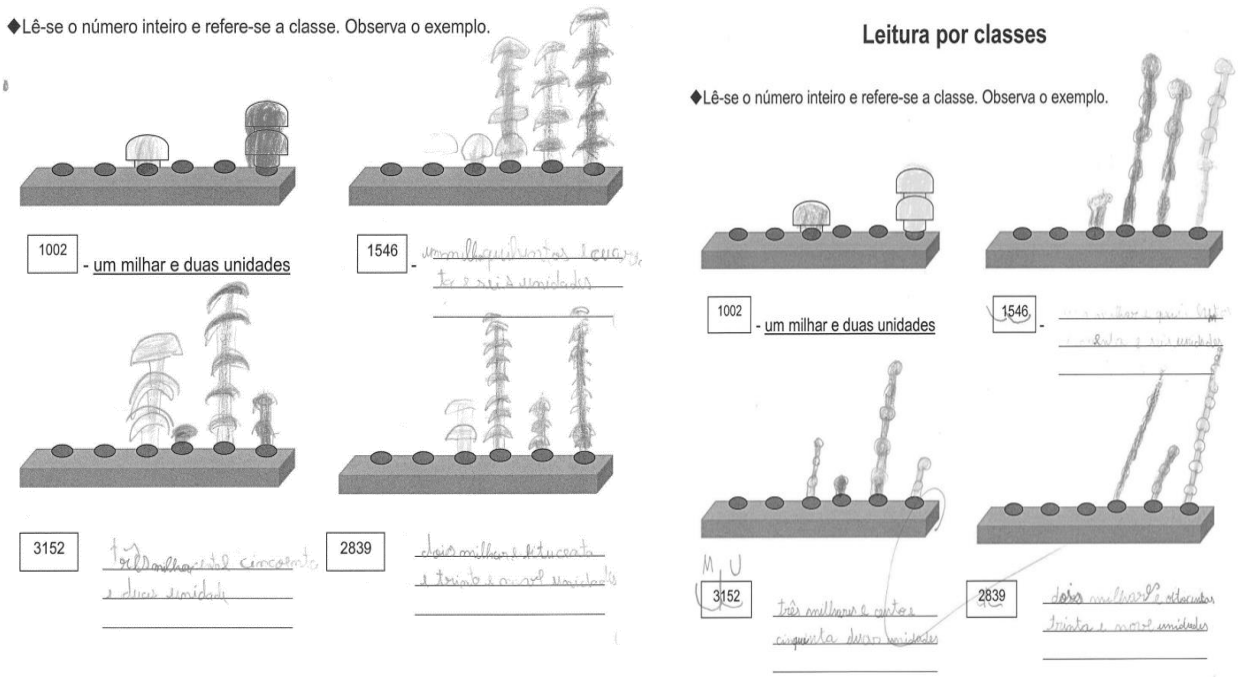
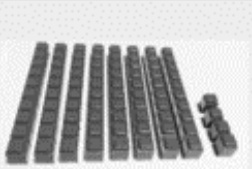
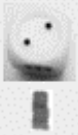


Figure 11 Task Solving by two students from Luisa class

Problem Solving using manipulative Mathematical material.

Grupos	Pontos que tinham	Pontos saídos	Pontos com que ficaram
Grupo A			
	80 + 4 = 84	6	84 + 6 =_____
Grupo B			
	90 + 8 = 98	2	98 + 2 = _____

Adaptado de Mota, S., Lima, E., Ferronilho, F., Santos, M., Semião, N. & Pedroso, N. (2018). TOP10 Matemática, 2.º ano. Porto: Porto Editora

Figure 12 Task instructions

Irina explored *Base 10* and, *Embossed Dice* and Task instructions in *Braille* Code to solve the problem.

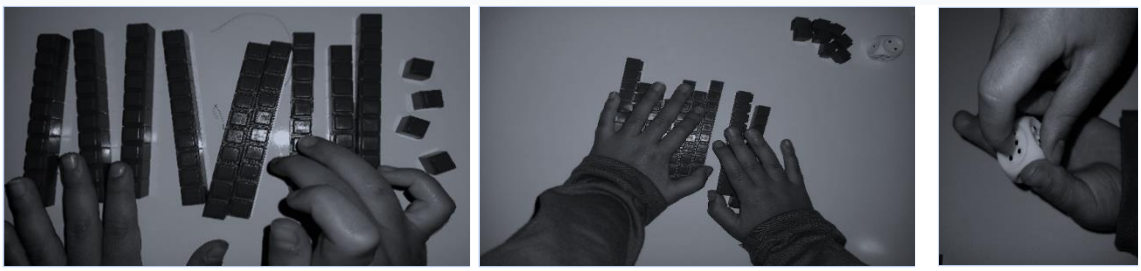
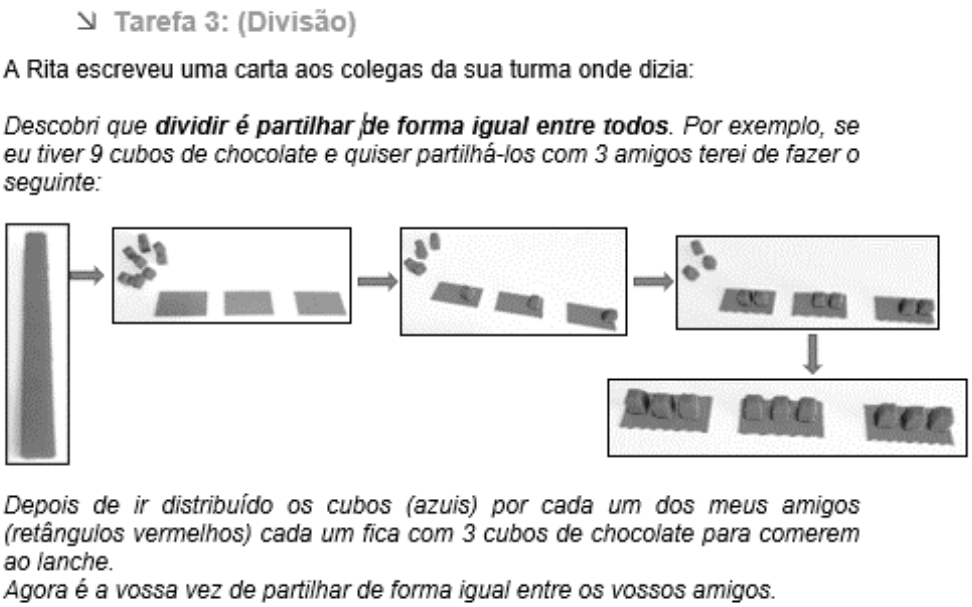


Figure 13 Task exploration using manipulative resources (Irina)

Example of a Task question using a procedure to introduce the notion of division into equal parts (Task 3 Division)



↘ Tarefa 3: (Divisão)

A Rita escreveu uma carta aos colegas da sua turma onde dizia:

Descobri que ***dividir é partilhar de forma igual entre todos****. Por exemplo, se eu tiver 9 cubos de chocolate e quiser partilhá-los com 3 amigos terei de fazer o seguinte:*

Depois de ir distribuído os cubos (azuis) por cada um dos meus amigos (retângulos vermelhos) cada um fica com 3 cubos de chocolate para comerem ao lanche.
Agora é a vossa vez de partilhar de forma igual entre os vossos amigos.

Figure 9 Task 3

Examples of responses from sighted students that divide 25 chocolate cubes between 5 friends:

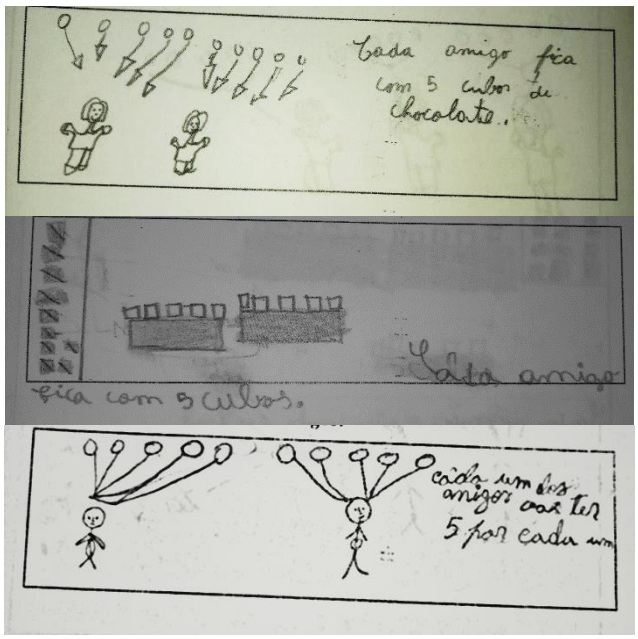


Figure 10 Answer examples

The resource used by students with blindness was the Multiplane.

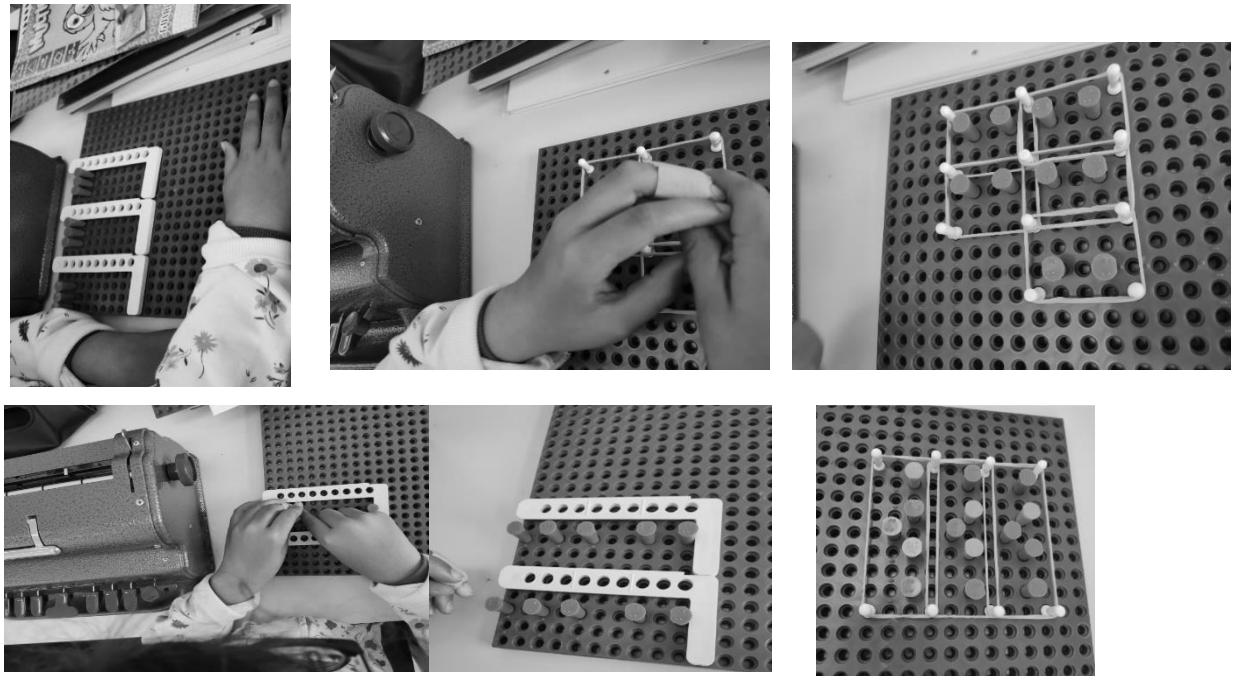


Figure 11 Procedure used to solve Task 3 using Multiplane (Luísa)

School 2 opted to use the Principles of UDL within the classroom context.

Methodology and Resources	**Obstacles encountered by some pupils**		**Abilities and Likes**	**Solutions to eliminate obstacles encountered in…**		
	Pupils	**Identifying Obstacles**		**… Means of Engagement (Motivation)**	**… Means of Representation**	**… Means of Action and Expression**
Methodology: Methodology of active work, centred around the student through heterogeneous pair work called Efficient Teams (pair or small group) **Resources:** Task questions Geometrical Solids Multiplane Cubaritm Perkins *Brailler* Machine Ruler, drawing material (pencil and rubber) TV monitor amplifier	Class ALANA (13 years old) Irina	Difficulty in acquiring the skills need in Problem Solving and Mathematical Communication (Reasoning). Concentration problems. Reasoning, Mathematical Communication Problem Solving and Low Self-esteem Easily gives up Portuguese Language Skills Undestanding questions. Note: **Universal Measures** (Curricular Adaptations) Obstacles of sensory nature (Blind student) Note: **Selective Measures** (Psych pedagogical support given by	Cooperative and participating class Iniciative and cooperation Spacio-visual Body-cinestesy Persistent Cooperation Body-cinestesy Linguistics	To motivate pupils the Song even and odd numbers by Luisa Ducla Soares will be used. **Task 1** instructions include the lyrics of the Song so that pupils can sing alone. **Task 2** question presents images using Base 10 manipulative resources and an Embossed Dice	The teacher will hand out instructions for the two tasks. The instructions are different and there are three different versions. - Instructions explaining how to carry out the activities, without information in bold nor pictures. - Instructions explaining how to carry out the activities, with important information in bold. - Instructions explaining how to carry out the activities in *Brailler* code using *Braille* Mathematical Wiriting. The instructions of **Task 1** include the lyric of the Song and the pupils can write directing on the task papers. This allows the pupils to describe his/her reasoning and conclusions.	Pupils can solve **Task 1** using ink or *Braille*. This Task contains helping instructions. However, they require the use of structured thoughts of the inferior and superior Bloom's Taxonomy. **Task 1** was elaborated to that all pupils could complete most or all the questions. **Task 2** can be solved using paper and pencil or other manipulative resources. **Task 3** present a picture that shows what the pupils must do. They can choose how they will carry out this task. In the case of the blind pupil, Multiplane will be used as manipulative resource. The pupil will decide how to layout the final

Self - evaluation using SMILES		special needs teacher and technical assistance)		**Task 3** question presents images using Base 10 manipulative resources The manipulative material will be given to each pair of pupils as a way of motivating in/her into participating in the tasks.	**Task 2** aims at interpreting the information given in the questions. These questions contain visual information that help understand the situation described. Apart from that, each pair will receive manipulative resources (Base 10, dice or embossed dice)  **Task 3** aims elaborating a procedure to begin division problems. The question contains illustrations that show the division process in equal parts. The pupil is asked to do the same in other contexts.	solution according to her multiple learning. The tasks used in this classroom illustrate how Differentiated Pedagogy is used in a classroom context. The way the questions were elaborated conjugated Bloom's Taxonomy and Gardner's Multiple Intelligences (Bloom-Gardner Matrix).

Table 13 Example of the Lesson Plan UDL

Final Conclusions

This research work corroborates the results of the Corso & Dornelas (2015) study, as it defends a correlation between the difficulty in reading and Mathematics. The authors defend the need to promote strategies that will stimulate the phonetic process as well as the sense of numbers in a classroom situation.

In general terms, students who have already been diagnosed with learning disabilities, despite enjoying support measures for learning and inclusion, were those who showed worse results in the tasks given them.

The study of the results reinforced the need to apply Active Methodologies centered around the student within the classroom context.

The study of the present work made it clear that it is possible to work in a classroom in an inclusive teaching environment. Moreover, it is possible to include blind students in the class as long as teachers plan according to the models presented in the Integrating Table adapted to the students' specificities (UDL and Bloom-Gardner Matrix).

The teacher/student communication is simplified with the use of blind based resources.

Communication using specific Graphologies is not understood by the headteacher of the class but is by the special needs teacher who can translate into ink these registers making written communication accessible to the headteacher.

In the case of the two students with blindness, the teacher who works on skills in the area of *Braille* Literacy has a prominent role in the evolution of their knowledge, because the «*didática do sistema Braille e dos meios literácitos complementares, numa dimensão tifloinclusiva desejável, já deveria ser objeto de estudo obrigatório em todos os curricula profissionais e de investigação nas áreas das ciências da comunicação, da informação e da educação, sendo uma matéria ética e cívica, educomunicativa, pedagógica e cultural a passar a estar efetivamente presente no desenvolvimento biopsicossocial e humano de todos os cidadãos, desde que nascem*.» (Guerreiro, 2021:46).

Ad sumam, «*um professor com Perfil Inclusivo trabalha com os pais e as famílias dos alunos, sem descurar a cooperação com outros profissionais da educação. A mobilização de saberes, experiências e recursos de forma conjunta para a promoção do desenvolvimento integral da criança será um objetivo primordial*.» (Brazileiro, 2021:57) Collaboration between teachers in and out of the classroom context is fundamental to a healthy inclusive education.

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